Papers written by Australian Maths Software

SEMESTER ONE
YEAR 12
MATHEMATICS SPECIALIST
REVISION 1
UNIT 3
2016
Section Two
(Calculator–assumed)

Name: ________________________________

Teacher: ________________________________

TIME ALLOWED FOR THIS SECTION

Reading time before commencing work:	10 minutes
Working time for section:	100 minutes

MATERIAL REQUIRED / RECOMMENDED FOR THIS SECTION

To be provided by the candidate
Standard items*:* pens, pencils, pencil sharpener, highlighter, eraser, ruler.

Special items: drawing instruments, templates, notes on up to two unfolded sheet of A4 paper, and up to three calculators approved for use in the WACE examinations.

IMPORTANT NOTE TO CANDIDATES
No other items may be taken into the examination room. It is your responsibility to ensure that you do not have any unauthorised notes or other items of a non–personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor before reading any further.

To be provided by the supervisor
Question/answer booklet for Section Two.
Formula sheet retained from Section One.

Structure of this examination

	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One Calculator—free	**5**	**5**	**50**	**50**	35
Section Two Calculator—assumed	11	11	100	100	**65**
			Total marks	150	100

Instructions to candidates

1. The rules for the conduct of this examination are detailed in the Information Handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answer in the Question/Answer booklet.

3. You must be careful to confine your responses to the specific questions asked and to follow any instructions that are specific to a particular question.

4. Spare pages are provided at the end of this booklet. If you need to use them, indicate in the original answer space where the answer is continued i.e. give the page number.

5. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you do not use pencil, except in diagrams.

7. The Formula Sheet is not to be handed in with your Question/Answer booklet.

6. (6 marks)

Given $\boldsymbol{v}(t) = (2cos(2t))\boldsymbol{i} + (-sin(t))\boldsymbol{j}$

(a) find the expressions for $\boldsymbol{r}(t)$ and $\boldsymbol{a}(t)$ given $\boldsymbol{r}(\pi) = -\boldsymbol{j}$. (3)

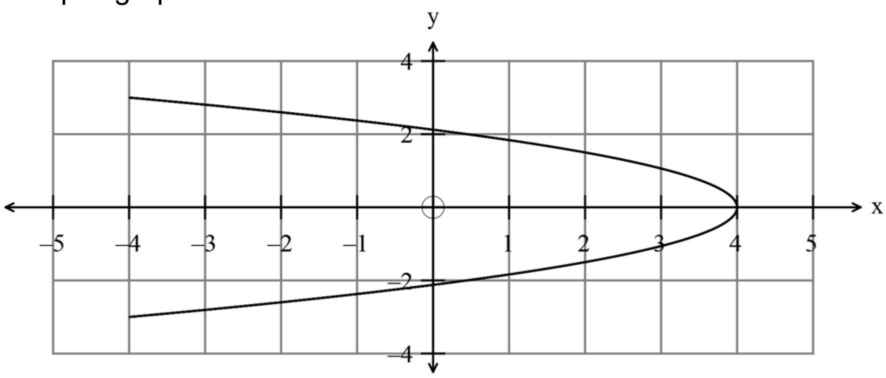
(b) determine $\boldsymbol{r}\left(\frac{3\pi}{2}\right)$ and $\boldsymbol{v}\left(\frac{3\pi}{2}\right)$. (2)

(c) show that $4\boldsymbol{r}(t) + \boldsymbol{a}(t) = 3cos(t)\boldsymbol{j}$. (1)

7. (20 marks)

The position vector of a particle at time t is given by $\boldsymbol{r}(t) = (4\cos(2t))\boldsymbol{i} + (3\sin(t))\boldsymbol{j}$.

The relationship is graphed below:



(a) Determine the position of the particle at $t = 0$ and indicate the direction of travel of the particle on the diagram. (3)

(b) Find an expression for the velocity of the particle and sketch the velocity vector on the graph for $t = 0$. (4)

(c) Find the expression for the acceleration of the particle. (2)

(d) Determine the position of the particle when $\boldsymbol{a}(t) = \begin{pmatrix} 16 \\ -3 \end{pmatrix}$. (2)

(e) Determine whether or not $\boldsymbol{a}(t)=k\,\boldsymbol{r}(t)$. (2)

(f) Determine the times, if any, on $[0,2\pi]$ when $\boldsymbol{a}(t)=\boldsymbol{0}$. (4)

(g) Convert the position vector $\boldsymbol{r}(t)=\left(4\cos(2t)\right)\boldsymbol{i}\ +\ \left(3\sin(t)\right)\boldsymbol{j}$ into the corresponding Cartesian equation. (3)

8. (10 marks)

(a) Find the vector equation of the plane that contains the points $A(3, 4, 0)$, $B(4, -3, 0)$ and $C(0, 0, 5)$. (2)

(b) Find the Cartesian equation of the sphere containing points $A(3, 4, 0)$, $B(4, -3, 0)$ and $C(0, 0, 5)$ by inspection. (2)

(c) A cricket ball was hit from $C(-3,5,2)$ with a velocity of $\begin{pmatrix} 4 \\ 3 \\ -1 \end{pmatrix}$ m/s.

Paul, at $P(3,4,2)$ leapt towards the ball with a velocity of $\begin{pmatrix} 1 \\ 3.5 \\ -1 \end{pmatrix}$ m/s.

(i) How many seconds will it take for the ball to hit the ground? (2)

If Paul reaches the ball, he will catch it.

(ii) Show that Paul catches the ball at ground level. (3)

Paul's Dad was telling the story and was inclined to exaggerate.

(iii) How far did he suggest Paul had leapt? (1)

9. (6 marks)

(a) (i) Find the vector equation of the line that contains the point $(2, 1, 3)$ and is parallel to the line MN where M has coordinates $(3, 4, 5)$ and N has coordinates $(-1, -2, -3)$. (2)

(ii) Find the corresponding Cartesian equation of the line. (2)

(b) Determine whether or not the lines L_1 *and* L_2 intersect. If they do, find the point of intersection.

$$L_1\text{: } \boldsymbol{r}_1(t) = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + t\begin{pmatrix} -1 \\ 0 \\ -1 \end{pmatrix}$$

$$L_2\text{: } \boldsymbol{r}_2(s) = \begin{pmatrix} 0 \\ 2 \\ 2 \end{pmatrix} + s\begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix}$$

(2)

10. (9 marks)

(a) Use the vector product to find the area of the triangle ABC given the points $A(1,2,3)$, $B(2,4,-1)$ and $C(3,-2,-3)$. (4)

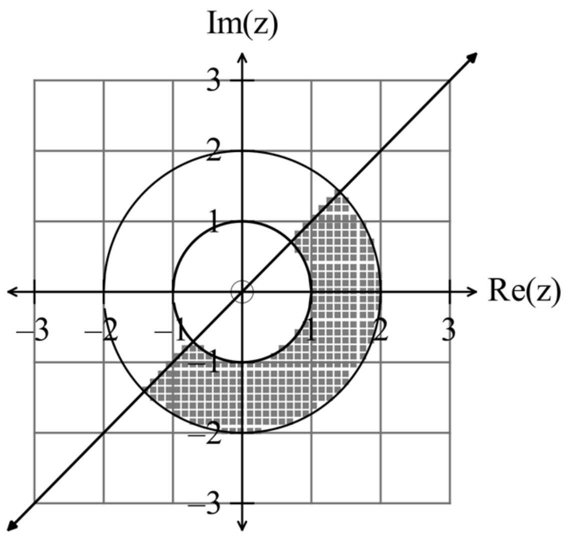
(b) Find the ratio $AB : AC$. (2)

(c) The midpoints of AB and AC are P and Q.
Prove PQ is parallel to BC. (3)

11. (13 marks)

(a) If $x = cis\left(\frac{\pi}{4}\right)$, $y = 1 - i$ and $z = 1 + \sqrt{3}i$, simplify $\frac{(xy)^3}{\sqrt{z}}$. (4)

(b) Describe fully the shaded region graphed on the set of axes below. (3)



(c) (i) By letting $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$ show that $\overline{z_1 z_2} = \overline{z_1} \times \overline{z_2}$. (3)

(ii) If $z = x + iy$ then find x and y if $z(1+i) + \bar{z}(1-i) + 2z = 10 - 2i$ (3)

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12. (6 marks)

A naughty Year 10 boy, Tom, kicked a football directly towards his English classroom window during his lunch break. Tom kicked the ball with a velocity of 20 m/sec at an angle of 60^0 from a height one metre off the ground from 50 metres away.
The window was at a height of 5.6 metres above the ground.
HINT: $\boldsymbol{a}(t) = -9.8\,\boldsymbol{j} \;\; m/s^{-2}$

(a) Is it possible that Tom kicked the ball through the window? (5)

The deputy was on lunch duty and left the English building three seconds after Tom kicked the ball

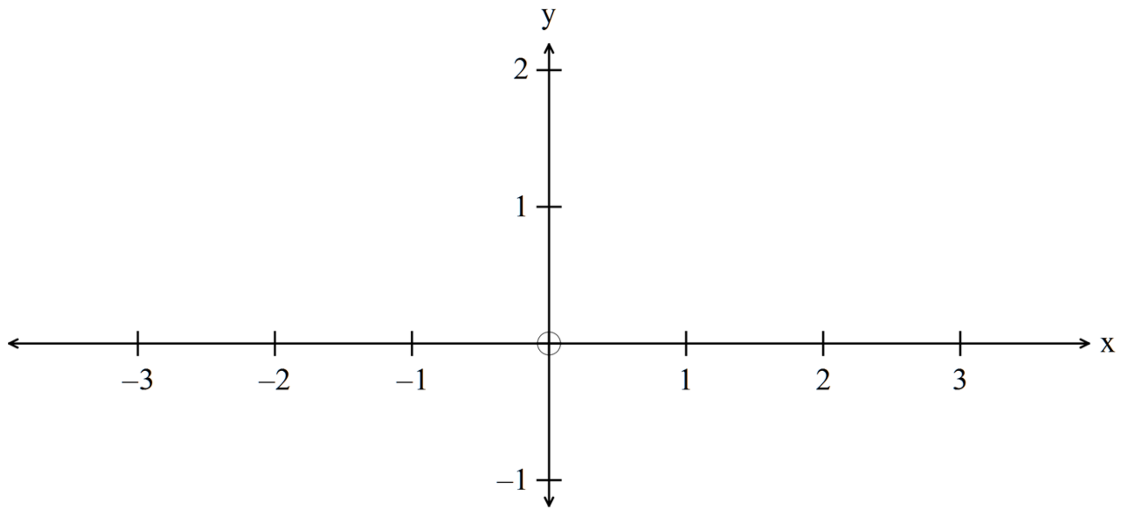
(b) Is it possible the deputy saw the ball in flight? (1)

13. (14 marks)

Given the functions $f(x) = sin(x)$, $g(x) = x^2$ and $h(x) = \sqrt{x}$

(a) (i) find $y = g(f(x))$. (1)

(ii) sketch $y = g(f(x))$ on the set of axes below. (3)



(iii) determine the maximum value of a such that $y = g(f(x))$ is a one to one function for $0 \le x \le a$. (2)

(iv) sketch the inverse of $y = g(f(x))$ on the set of axes above. Label each graph. (2)

(b) Given $h(x) = \sqrt{x}$,

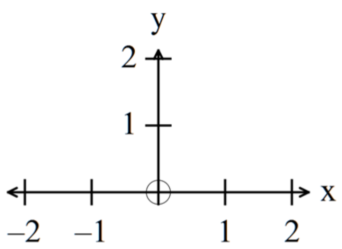
(i) determine the equation of the inverse function $y = h^{-1}(x)$ (1)

(ii) write down the range of the inverse function h^{-1} . (1)

(c) Find $y = g(h(x))$ and hence determine where the function is defined on $[-2\pi, 2\pi]$.

(2)

(d) (i) Sketch $y = |x|$ on the set of axes below. (1)



(ii) Explain why the function $y = |x|$ has no inverse. (1)

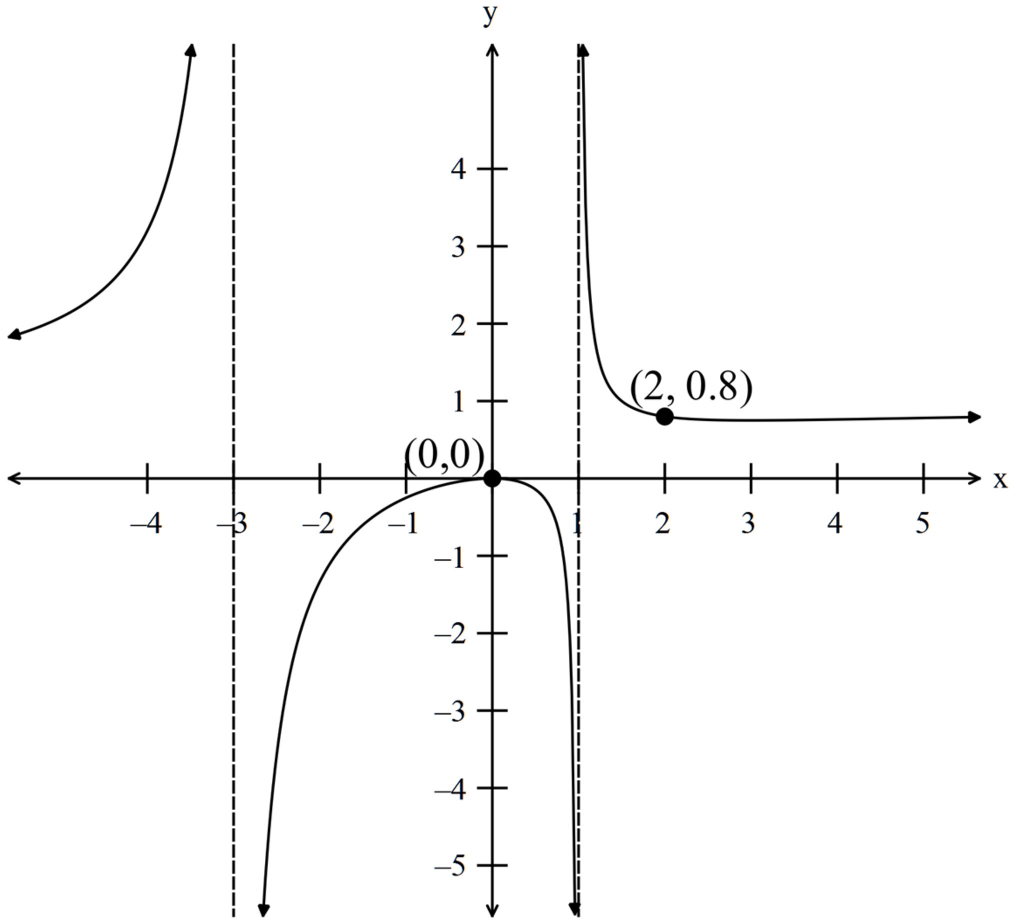
14. (7 marks)

(a) Find x such that $|x+1|-1>0$. (2)

(b) Solve $|1+x|-1=|x-1|$. Show all working. (5)

15. (4 marks)

Determine the equation of the function graphed below: (4)



16. (5 marks)

(i) Use your calculator to complete the following table. (2)

n	$(1-i)^n$
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	

(ii) Comment on any patterns found. (3)
NB Much further investigation is required to determine all the patterns.

END OF SECTION TWO